2.40.1. Derived Rule Problems

1. Provide a deduction for the following argument, using only **~–**, and **~+**.

$$\begin{array}{l} 1.\ \ P \\ \hline \therefore\ P \end{array}$$

2. Provide a deduction for the argument in Problem 1 using only **∨+**, **∨–**, and **ID**.

3. Provide a deduction for the following argument, using only **∨+**, **∨–**, and **ID**.

$$\begin{array}{l} 1.\ {\sim}{\sim}P \\ \hline \therefore\ P \end{array}$$

4. Provide a deduction for the argument in Problem 3 using only R and **ID**.

5. Provide a deduction for the following argument, using only R and **ID**.

$$\begin{array}{l} 1.\ \ P \\ \hline \therefore\ {\sim}{\sim}P \end{array}$$

6. Suppose we use a variant deductive system – call it "**DS1**" – consisting of the Chapter Two deductive system, but with Inward DM in place of ∨**+** and ∧**+**.[1]

Inward DM (In DM)

$$\frac{\sim(\bullet \vee \blacktriangle)}{(\sim\bullet \wedge \sim\blacktriangle)} \qquad \frac{\sim(\bullet \wedge \blacktriangle)}{(\sim\bullet \vee \sim\blacktriangle)}$$

Show that **DS1** is deductively equivalent to the Chapter Two system, by constructing a **DS1** deduction for each of the following two arguments.

$$\frac{\begin{array}{l}1.\ \ P\\ 2.\ \ Q\end{array}}{\therefore\ (P \wedge Q)} \qquad \frac{1.\ \ P}{\therefore\ (P \vee Q)}$$

7. **DS2** is another variant deductive system, consisting of the Chapter Two deductive system, but with **Outward DM** in place of ∨**–** and ∧**–**.

Outward DM (Out DM)

$$\frac{(\sim\bullet \wedge \sim\blacktriangle)}{\sim(\bullet \vee \blacktriangle)} \qquad \frac{(\sim\bullet \vee \sim\blacktriangle)}{\sim(\bullet \wedge \blacktriangle)}$$

Show that **DS2** is deductively equivalent to the Chapter Two system, by constructing a **DS 2** deduction for each of the following two arguments.

$$\frac{\begin{array}{l}1.\ \ (P \vee Q)\\ 2.\ \ \sim P\end{array}}{\therefore\ Q} \qquad \frac{1.\ \ (P \wedge Q)}{\therefore\ P}$$

[1] As Problems 4 and 5 show, we could also strip out ~**+** and ~**–** in this system without loss of deductive power.

8. Semantics and deduction show the following rule to be valid.[2]

Double Disjunction (DD)

(● ∨ ▲)	(▲ ∨ ●)
(● ∨ ~▲)	(~▲ ∨ ●)
●	●

Provide a deduction for the following two arguments using only **DD** and **∨+** (but without using **ID**).

1. (P ∨ Q) 2. ~P	1. P
∴ Q	∴ P

9. Provide a deduction for the following two arguments using only **DD**, **∨+**, and **ID**.

1. ~~P	1. P
∴ P	∴ ~~P

(*Hint: for the right argument, adapt your deduction of the left argument to get "~P" from "~~~P". Alternate route: combine Problem 5 with the right half of Problem 8.*)

[2] This argument form was encountered earlier, in 2.38.1 Problem A.

10. Recall that we earlier examined the behavior of **anti-valuation sentences** – disjunctions whose parts are basics, with each sentence letter used appearing just once in the sentence.[3] For example, the following are the family of anti-valuation sentences built from sentence letters {P, Q}.

{P, Q} Anti-Valuation Sentences

1. $(P \lor Q)$	2. $(P \lor \sim Q)$	3. $(\sim P \lor Q)$	4. $(\sim P \lor \sim Q)$

It was noted, in particular, that to build a **contradiction** from anti-valuation sentences we must conjoin **all** the anti-valuation sentences in that family.[4]

But with Double Disjunction in hand we now note how pairs of anti-valuation sentences entail a basic from this family – either "P," "Q," "~P," or "~Q".

1. $(P \lor Q)$	1. $(P \lor Q)$	3. $(\sim P \lor Q)$	2. $(P \lor \sim Q)$
2. $(P \lor \sim Q)$	3. $(\sim P \lor Q)$	4. $(\sim P \lor \sim Q)$	4. $(\sim P \lor \sim Q)$
$\therefore$ P	$\therefore$ Q	$\therefore$ ~P	$\therefore$ ~Q

Which of these conclusions, if conjoined together, yield a contradiction, and which anti-valuation sentences get us to those conclusions? Does the pattern hold for the anti-valuation sentences in the {P, Q, R} family?[5] (Which premises are required to entail "P?" "~P"? "Q"? "~Q"? "R"? ~R"?)

{P, Q, R} Anti-Valuation Sentences

1. $(P \lor Q \lor R)$	2. $(P \lor Q \lor \sim R)$	3. $(P \lor \sim Q \lor R)$	4. $(P \lor \sim Q \lor \sim R)$
5. $(\sim P \lor Q \lor R)$	6. $(\sim P \lor Q \lor \sim R)$	7. $(\sim P \lor \sim Q \lor R)$	8. $(\sim P \lor \sim Q \lor \sim R)$

How does this explain which anti-valuation sentences from a family are needed to yield a contradiction?

[3] In 2.26 §2.
[4] In 2.26 §4.
[5] Feel free to change the order of the parts in a disjunction, to fit the general pattern of Double Disjunction from the previous page.

Semantics and deduction also show the following rule to be valid.[6]

Generalized Double Disjunction (GDD)

(● ∨ ▲)	(▲ ∨ ●)
(● ∨ ♥)	(♥ ∨ ●)
~(▲ ∧ ♥)	~(▲ ∧ ♥)
●	●

11. Provide a deduction for the following argument using only ∨+, **ID**, and **GDD**.

1. P
2. Q

∴ (P ∧ Q)

12. Provide a deduction for the following arguments using only R, ∧–, **ID**, and **GDD**.

1. (P ∨ Q)
2. ~P

∴ Q

(*Hint: first get "~(P ∧ P)" to use in a mutant form of the right version of GDD.)*

[6] This is a variant on the rule of **Separation of Cases** (or **Disjunctive Dilemma**) discussed in 3.14.1. DD takes for granted a particular form of the third premise of GDD (as Problem 13 illustrates): a negated conjunction of the form ~(● ∧ ~●).

13. Provide a deduction for the following argument using only R, **∧–**, **ID**, and **GDD**.

1. $(P \vee Q)$
2. $(P \vee \sim Q)$

$\therefore$ P